Problem 1

a) P(A|B) = P(A∩B) / P(B) = 1/3

b) P(B|A) = P(A∩B) / P(A) = 1/5

c) P(A∪B) = P(A) + P(B) – P(A∩B) = 0.7

P(A|A∪B) = P(A∩(A∪B)) / P(A∪B) = P(A) / P(A∪B) = 5/7

d) P(A|A∩B) = P(A∩(A∩B)) / P(A∩B) = P(A∩B) / P(A∩B) = 1

e) P(A∩B|A∪B) = P(A∩B) / P(A∪B) = 1/8

Problem 2

a) p(E1) = 1/2, p(E2) = 1/2, p(E1∩E2) = 1/4 = p(E1)·p(E2), 是

b) p(E1) = 1/2, p(E2) = 3/8, p(E1∩E2) = 1/8 ≠ p(E1)·p(E2), 不是

c) p(E1) = 1/2, p(E2) = 3/8, p(E1∩E2) = 1/8 ≠ p(E1)·p(E2), 不是

Problem 3

a) E1: 5名志愿者血液中均含恒河因子, P(-E1) = 1 – P(E1) = 1 – 80%^5 = 0.67232

b) E2: 5名志愿者血液中均含恒河因子, P(-E2) = 1 – P(E2) = 1 – 80%^5 = 0.67232

c) E3: n名志愿者中至多4名血液含Rh因子, P(-E3) = 1 – P(E3)

P(E3) = ∑4, k=0 C(n, k)×0.8^k×0.2^(n-k) < 1 – 90% = 0.1, 又n∈N

代入数据得n=7时P(E3) > 0.1, n=8时P(E3) < 0.1 最少志愿者人数是8.

Problem 4

a) C(n, 2) (n-2)! / n! = 1/2

b) C(n, 4) C(4, 2) (n-4)! / n! = 1/4

c) C(n, 2) (n-2)! / n! = 1/2

d) 2 C(n, 3) (n-3)! / n! = 1/3

e) (n-1)! / n! = 1/n

Problem 5

a) 0.97×0.04 / (0.97×0.04 + 0.02×0.96) ≈ 0.669

b) 1 – 0.669 = 0.331

c) 0.03×0.04 / (0.03×0.04 + 0.98×0.96) ≈ 0.001

d) 1 – 0.001 = 0.999

Problem 6

E: Remesh迟到, A: Remesh骑自行车, B: Remesh开车, C: Remsesh坐公共汽车.

则有P(E|A) = 5%, P(E|B) = 50%, P(E|C) = 20%.

a) P(A) = P(B) = P(C) = 1/3

P(B|E) = P(E|B)·P(B) / P(E|A)·P(A) + P(E|B)·P(B) + P(E|C)·P(C) = 2/3

b) P(A) = 60%, P(B) = 30%, P(C) = 10%

P(B|E) = P(E|B)·P(B) / P(E|A)·P(A) + P(E|B)·P(B) + P(E|C)·P(C) = 3/4

Problem 7

∑4 i=0 C(9, 2i) · (1/2)^9 = (1+36+126+84+9)·(1/2)^9 = 256 / 2^9 = 1/2

Problem 8

假设100个座位随机选择n个座位，所选的连续座位对的期望是P(n)

P(1) = 0, P(2) = 1×C(99, 1)/C(100, 2) = 1/50

P(3) = 2×C(98, 1)/C(100, 3) + 1×C(98, 2)/C(100, 3) = 101/3300

P(n) = ∑n-1 i=1 i·C(100-n+1, n-i)/C(100, n)

P(25) = ∑24 i=1 i·C(76, 25-i) / C(100, 25) 25/100×24/99×99 = 6.

Problem 9

npq = 10×(1/6)×(1-1/6) = 25/18

Problem 10

E: 它不会被拒绝(没有观察到次品), P(E) = C(16, 5) / C(20, 5) = 91/323

P(-E) = 1 – P(E) = 1 – 91/323 = 232/323 ≈ 0.72

样本大小为5的采样中次品的预期数量:

∑4 i=0 i·C(4, i)C(16, 5-i) / C(20, 5) = 15504/15504 = 1

样本大小为5的采样中次品数量的方差是多少

∑4 i=0 (i-1)^2·C(4, i)C(16, 5-i) / C(20, 5) = 13152/15504 ≈ 0.85